Delta link-homotopy on spatial graphs*

Ryo Nikkuni†

Two spatial embeddings of a graph are said to be *delta edge-homotopic* (resp. *delta vertex-homotopic*) if they are transformed into each other by *self delta moves* (resp. *quasi adjacent-delta moves*) and ambient isotopies. Our purpose in this talk is to explain the recent topics related to delta edge (vertex)-homotopy on spatial graphs. We refer the audience to [27, 28, 29] for the content of this talk and [31, 32, 33] for their outlines in Japanese. References contain not only articles which are cited in this note but also articles which will be cited in this talk.

0. Equivalence relations on spatial graphs

Let G be a finite graph which does not contain free vertices. We consider G as a topological space in the usual way. An embedding $f : G \to S^3$ is called a *spatial embedding of* G or simply a *spatial graph*.

Definition 0.1. ([42]) Two spatial embeddings f and g of a graph G are said to be
(1) *ambient isotopic* if there is a level preserving locally flat embedding $\Phi : G\times I \to S^3\times I$ between f and g.
(2) *cobordant* if there is a locally flat embedding $\Phi : G \times I \to S^3 \times I$ between f and g.
(3) *isotopic* if there is a level preserving embedding $\Phi : G \times I \to S^3 \times I$ between f and g.
(4) *I-equivalent* if there is an embedding $\Phi : G \times I \to S^3 \times I$ between f and g.
(5) *edge-homotopic* if they are transformed into each other by *self crossing changes* and ambient isotopies, where self crossing change is a crossing change on the same spatial edge.

We refer the audiences to [42] for the precise definitions. We remark here that edge-homotopy is a natural generalization of *link-homotopy* in the sense of J. Milnor [16].

A graph G is said to be *planar* if there exists an embedding of G into S^2. It is known that an embedding of G into $S^2 \subset S^3$ is unique up to ambient isotopy [14].

Definition 0.2. A spatial embedding of a planar graph G is said to be
(1) *trivial* if it is ambient isotopic to an embedding of G into $S^2 \subset S^3$.
(2) *slice* if it is cobordant to the trivial spatial embedding.

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1. Delta edge (vertex)-homotopy

A *delta move* is a local move on spatial graphs as illustrated in Figure 1.1 (1). It is well known that the delta move is an unknotting operation [15, 21], namely if $G \approx S^1$ then any two knots can be transformed into each other by delta moves and ambient isotopies.

Definition 1.1. ([27]) (1) A *self delta move* is a delta move on the same spatial edge (see Figure 1.1 (2)). Two spatial embeddings of a graph are said to be *delta edge-homotopic* if they are transformed into each other by self delta moves and ambient isotopies.
(2) A *quasi adjacent-delta move* is a delta move on exactly two adjacent spatial edges (see Figure 1.1 (3)). Two spatial embeddings of a graph are said to be *delta vertex-homotopic* if they are transformed into each other by quasi adjacent-delta moves and ambient isotopies.
(3) A spatial embedding of a planar graph is said to be *delta edge* (resp. *vertex*)-*homotopically trivial* if it is delta edge (resp. vertex)-homotopic to the trivial one.

These are natural generalizations of *delta link-homotopy* (or a *self delta-equivalence*) on links that is an equivalence relation generated by delta moves on the same component [38, 39, 26, 25, 22, 23, 24]. It can be easily seen that any of the local knots attached to a spatial edge can be undone up to delta edge-homotopy.

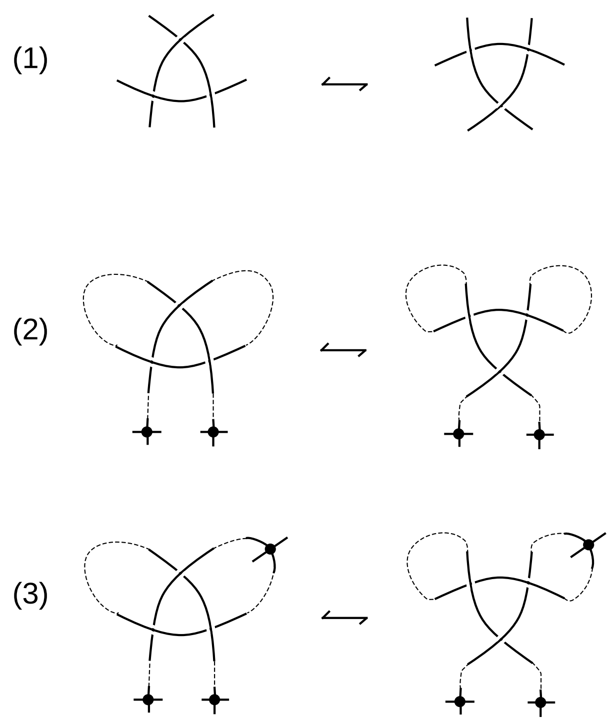
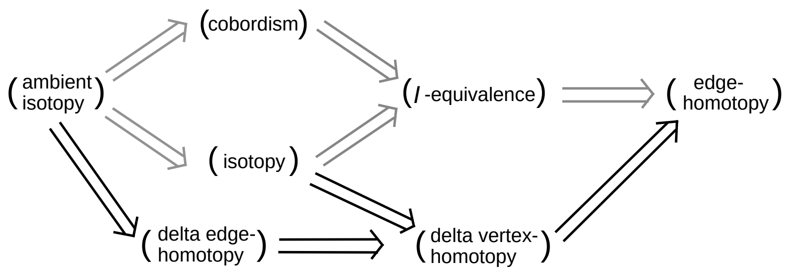


Fig. 1.1.

2. Relation to the other equivalence relations

First we investigate how strong is delta edge (resp. vertex)-homotopy and decide whether or not a graph has a delta edge (resp. vertex)-homotopically non-trivial spatial embedding.

Theorem 2.1. ([27])



Moreover these equivalence relations are different equivalence relations. □

We note that the gray implications have already proved in [42, FUNDAMENTAL THEOREM]. We have the following as a corollary of Theorem 2.1 and [42, Theorem B].

Corollary 2.2. ([27]) *The following conditions are mutually equivalent.*
(1) *Any two spatial embeddings of G are isotopic.*
(2) *Any two spatial embeddings of G are I-equivalent.*
(3) *Any two spatial embeddings of G are delta vertex-homotopic.*
(4) *Any two spatial embeddings of G are edge-homotopic.*
(5) *A graph G is a generalized bouquet, namely G does not contain a subgraph that is homeomorphic to the graphs $S^1 \amalg S^1$, K_4 or D_3 as illustrated in Figure 2.1.* □

On the other hand, for delta edge-homotopy we have the following.

Theorem 2.3. ([30]) *The following conditions are mutually equivalent.*
(1) *Any two spatial embeddings of G are delta edge-homotopic.*
(2) *A graph G does not contain a subgraph that is homeomorphic to the graphs θ or $S^1 \amalg S^1$ as illustrated in Figure 2.1.*
(3) *A graph G is homeomorphic to a bouquet B_m as illustrated in Figure 2.1.* □

3. Delta edge (resp. vertex)-homotopy invariants

To detect a delta edge (resp. vertex)-homotopically non-trivial spatial embedding of a graph, we construct some invariants. A *cycle* is a subgraph of G which is homeomorphic to S^1, and a *k-cycle* is a cycle which contains exactly k vertices. We denote the set of all cycles of G, the set of all cycles containing an edge e of G and the set of all

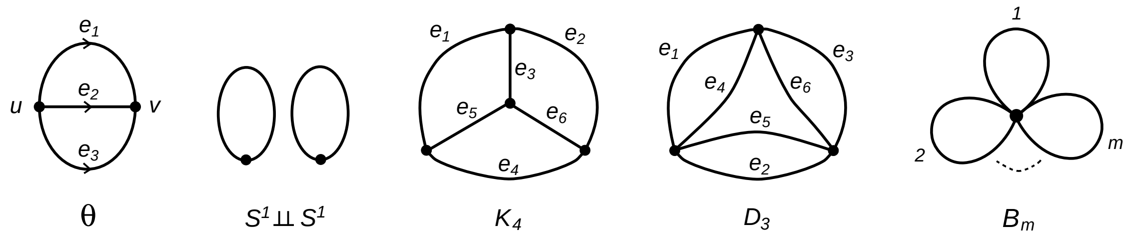


Fig. 2.1.

cycles containing edges e_1, e_2 of G by $\Gamma(G)$, $\Gamma_e(G)$ and $\Gamma_{e_1,e_2}(G)$, respectively. Let $\mathbf{Z}_m = \{0, 1, \ldots, m-1\}$ for a positive integer m and $\mathbf{Z}_0 = \mathbf{Z}$. A map $\omega : \Gamma(G) \to \mathbf{Z}_m$ is called a *weight* on $\Gamma(G)$. For an edge e and adjacent edges e_1, e_2 of G,

Definition 3.1. A weight $\omega : \Gamma(G) \to \mathbf{Z}_m$ is said to be
(1) *weakly balanced on* e if $\sum_{\gamma \in \Gamma_e(G)} \omega(\gamma) \equiv 0 \pmod{m}$.
(2) *weakly balanced on* e_1 *and* e_2 if $\sum_{\gamma \in \Gamma_{e_1,e_2}(G)} \omega(\gamma) \equiv 0 \pmod{m}$.

Definition 3.2. ([43]) A weight $\omega : \Gamma(G) \to \mathbf{Z}_m$ is said to be
(1) *balanced on* e if $\sum_{\gamma \in \Gamma_e(G)} \omega(\gamma)\gamma = 0 \in H_1(G; \mathbf{Z}_m)$.
(2) *balanced on* e_1 *and* e_2 if $\sum_{\gamma \in \Gamma_{e_1,e_2}(G)} \omega(\gamma)\gamma = 0 \in H_1(G; \mathbf{Z}_m)$.

For a weight $\omega : \Gamma(G) \to \mathbf{Z}_m$ and a spatial embedding f of G, we set

$$\tilde{\alpha}_\omega(f) \equiv \sum_{\gamma \in \Gamma(G)} \omega(\gamma) a_2(f(\gamma)) \pmod{m},$$
$$n_\omega(f) \equiv \frac{1}{18} \sum_{\gamma \in \Gamma(G)} \omega(\gamma) V^{(3)}_{f(\gamma)}(1) \pmod{m},$$

where $a_i(L)$ and $V_L^{(k)}(1)$ denote the i-th coefficient of the *Conway polynomial* and the k-th derivative at 1 of the *Jones polynomial*[1] $V_L(t)$ of a link L, respectively. We note that $(1/18)V_J^{(3)}(1)$ is an integer for any knot J.

Theorem 3.3. ([27]) (1) *If* ω *is weakly balanced on each of edges (resp. pair of adjacent edges) of* G*, then* $\tilde{\alpha}_\omega$ *is a delta edge (resp. vertex)-homotopy invariant.*
(2) *If* ω *is balanced on each of edges (resp. pair of adjacent edges) of* G*, then* n_ω *is a delta edge (resp. vertex)-homotopy invariant.* □

Example 3.4. Let f be a theta curve as illustrated in Figure 5.2 (1). We define a weight $\omega : \Gamma(\theta) \to \mathbf{Z}_2$ by $\omega(\gamma) = 1$ for any $\gamma \in \Gamma(\theta)$. Then it is easy to see that ω is weakly balanced on each of edges of θ. Then by a calculation we have that $\tilde{\alpha}_\omega(f) = 1$. So f is a delta edge-homotopically non-trivial.

[1] We calculate the Jones polynomial of a link by the skein relation $tV_{J_+}(t) - t^{-1}V_{J_-}(t) = (t^{-\frac{1}{2}} - t^{\frac{1}{2}})V_{J_0}(t)$.

By using n_ω-invariant, we can show that there exist infinitely many spatial embeddings of K_4 up to delta edge-homotopy which are mutually delta vertex-homotopic [27, Example 4.2]. Besides we can show that there exist infinitely many spatial embeddings of K_5 up to delta vertex-homotopy which are mutually edge-homotopic [27, Example 4.3]. We note that this is also an example of infinitely many spatial embeddings of K_5 up to isotopy which are mutually edge-homotopic.

Remark 3.5. If a weight ω is balanced on each of edges (resp. each pair of adjacent edges) of G, then our $\tilde{\alpha}_\omega$ coincides with the *α-invariant* α_ω [43] that is known as an edge (resp. vertex)-homotopy invariant of spatial graphs.

4. Edge-homotopy classification of spatial embeddings of K_4

According to Corollary 2.2, there exist two spatial embeddings of each of the graphs $S^1 \amalg S^1$, K_4 and D_3 that are not edge-homotopic. Moreover, it is well known that two spatial embeddings of $S^1 \amalg S^1$ are edge-homotopic if and only if they have the same *linking number* [16]. We classify spatial embeddings of K_4 up to edge-homotopy from a viewpoint of delta vertex-homotopy.

Definition 4.1. An *adjacent-delta move* is a delta move on exactly three adjacent spatial edges (see Figures 4.1). We say that two spatial embeddings of a graph are Δ-*homotopic* if they are transformed into each other by quasi adjacent-delta moves, adjacent-delta moves and ambient isotopies.

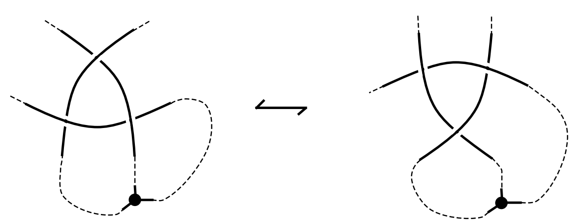


Fig. 4.1.

We define a weight $\omega : \Gamma(K_4) \to \mathbf{Z}$ by $\omega(\gamma) = 1$ if γ is a 3-cycle and $\omega(\gamma) = -1$ if γ is a 4-cycle. Then it is easy to see that ω is balanced on each of edges of G. We can also define a balanced weight $\omega : \Gamma(D_3) \to \mathbf{Z}$. Therefore $\alpha_\omega(f) = \tilde{\alpha}_\omega(f)$ for a spatial embedding f of K_4 (resp. D_3) is an edge-homotopy invariant (cf. Remark 3.5).

Theorem 4.2. ([28]) *Let G be a graph that is K_4 or D_3. Then two spatial embeddings f and g of G are Δ-homotopic if and only if $\alpha_\omega(f) = \alpha_\omega(g)$.* $\square$

In fact an adjacent-delta move on spatial embeddings of K_4 is always realized by quasi adjacent-delta moves. Therefore we have the following by Theorem 2.1 and Theorem 4.2.

Theorem 4.3. ([28]) *Let f and g be spatial embeddings of K_4. Then the following conditions are mutually equivalent.*
(1) *Two spatial embeddings f and g are Δ-homotopic.*
(2) *Two spatial embeddings f and g are delta vertex-homotopic.*
(3) *Two spatial embeddings f and g are edge-homotopic.*
(4) $\alpha_\omega(f) = \alpha_\omega(g)$. □

We remark here that the α-invariant of a spatial embedding f of K_4 can be interpreted as Milnor's *μ-invariant* [16] of an associated 3-component link of f. We also remark here that edge-homotopy classes of spatial embeddings of D_3 have not classified yet.

5. Delta edge-homotopy on theta curves

According to Theorem 2.3, there exist two spatial embeddings of each of the graphs $S^1 \amalg S^1$ and θ that are not delta edge-homotopic. Moreover, spatial embeddings of $S^1 \amalg S^1$ were classified completely up to delta edge-homotopy [24]. So we want to classify theta curves up to delta edge-homotopy next. For a graph θ, we put $\gamma_1 = e_2 \cup e_3$, $\gamma_2 = e_3 \cup e_1$ and $\gamma_3 = e_1 \cup e_2$.

Definition 5.1. ([11]) For a theta curve f, the *associated* 3*-component link* L_f is the boundary of an orientable surface S_f with zero *Seifert linking form* having f as a spine (see Figure 5.1). We order and orient $L_f = K_f^1 \cup K_f^2 \cup K_f^3$ so that K_f^i is freely homotopic to $f(e_{i+1}) - f(e_{i+2})$, where suffixes are taken modulo 3. We denote the sublink $K_f^{i+1} \cup K_f^{i+2}$ by $l_i(f)$ $(i = 1, 2, 3)$.

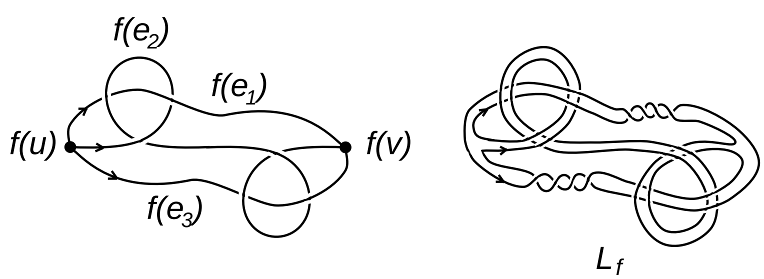


Fig. 5.1.

We note that if L is a 2-component link whose linking number is zero, then the *Sato-Levine invariant* $\beta(L)$ [37] coincides with $a_3(L)$ [1, 40]. It is known that $a_3(l_1(f)) = a_3(l_2(f)) = a_3(l_3(f))$ for any theta curve f [41] [8].

Definition 5.2. For a theta curve f, we define that $a_3(f) = a_3(S)$, where S is any 2-component sublink of L_f.

Theorem 5.3. ([29]) *Two theta curves f and g are delta edge-homotopic if and only if $a_3(f) = a_3(g)$.* $\square$

In fact we can see that $a_3(f) \equiv \sum_{i=1}^{3} a_2(f(\gamma_i)) \equiv \tilde{\alpha}_\omega(f) \pmod 2$, where $\omega : \Gamma(\theta) \to \mathbf{Z}_2$ is a weight as in Example 3.4. So our invariant $a_3(f)$ is finer than $\tilde{\alpha}_\omega(f)$.

Example 5.4. Let f be *Kinoshita's theta curve* as illustrated in Figure 5.2 (2). It is an example of an *almost unknotted* theta curve, namely each $f|_{\gamma_i}$ is a trivial knot ($i = 1, 2, 3$). By a calculation we have that $a_3(f) = 2$. So we have that Kinoshita's theta curve is delta edge-homotopically non-trivial.

In [29, Theorem 4.1] we give a calculation of $a_3(f)$ for almost unknotted theta curves by the third derivative at 1 of the *Kojima-Yamasaki η-function* [13]. We can show that for any integer m there exists a almost unknotted theta curve f such that $a_3(f) = 2m$ [29, Example 4.2] by using Wolcott's theta curves [48].

Definition 5.5. A theta curve f is called a *boundary theta curve* [34] if there are compact, connected and orientable surfaces S_1, S_2 and S_3 in S^3 such that $S_i \cap f(\theta) = \partial S_i = f(\gamma_i)$ $(i = 1, 2, 3)$ and $\mathrm{int} S_i \cap \mathrm{int} S_j = \emptyset$ $(i \neq j)$.

It is known that any 2-component boundary link is delta edge-homotopically trivial [39]. We have the similar result.

Corollary 5.6. ([29]) *Any boundary theta curve is delta edge-homotopicaly trivial.* $\square$

We note that he converse of Corollary 5.6 is not true. Besides we consider the relationship between cobordism on theta curves and delta edge-homotopy.

Corollary 5.7. ([29]) *Two cobordant theta curves are delta edge-homotopic. In particular, any slice theta curve is delta edge-homotopically trivial.* $\square$

We note that the converse of Corollary 5.7 is not true. We also note that cobordism on spatial graphs does not always imply delta edge-homotopy (cf. Theorem 2.1).

Let $\mathrm{DEH}(\theta)$ be the set of all delta edge-homotopy classes of theta curves. We denote the delta edge-homotopy class of a theta curve f by $[f]$. By Theorem 5.3 and Corollary 5.7, we can see the following.

Theorem 5.8. ([29]) *The set $\mathrm{DEH}(\theta)$ forms a group under the vertex connected sum. Moreover, we have an isomorphism $a_3 : \mathrm{DEH}(\theta) \xrightarrow{\cong} \mathbf{Z}$ by $a_3([f]) = a_3(f)$. A generator of $\mathrm{DEH}(\theta)$ is given by the theta curve f as illustrated in Figure 5.2 (1).* $\square$

Corollary 5.9. ([29]) *Delta edge-homotopy classes of almost unknotted theta curves form a subgroup of $\mathrm{DEH}(\theta)$ which is isomorphic to $2\mathbf{Z}$ under a_3. A generator of this subgroup is given by Kinoshita's theta curve.* $\square$

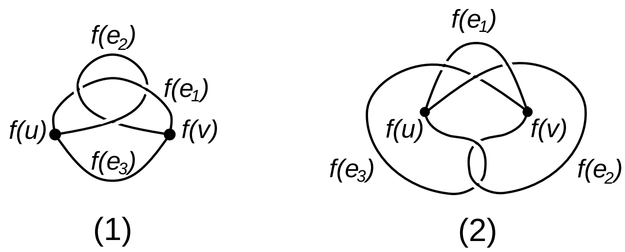


Fig. 5.2.

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Department of Mathematics, School of Education, Waseda University
Nishi-Waseda 1-6-1, Shinjuku-ku, Tokyo, 169-8050, Japan
nick@kurenai.waseda.jp