ON THE CONFIGURATION SPACE OF POINTS AND THE CASSON INVARIANT

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1. Introduction

Let M be an oriented closed homology 3-sphere, and $\lambda(M)$ Casson invariant of M. In this report, we construct some topological invariant $I(M)$ such that

- $I(M) = \lambda(M)$ (Theorem 1).
- $I(M) = -\frac{\operatorname{Sign} X_{f_M}}{8}$, where X_{f_M} is a certain 4-dimensional submanifold embedded in the two point configuration space of $M \setminus \{p\}$ (Theorem 2)

And we will also see outline of the proofs of these theorems (§ 5).

Roughly speaking, $\lambda(M)$ is defined by

$$\lambda(M) = \frac{1}{2}\#\frac{\operatorname{Hom}(\pi_1(M), SU(2))^{\mathrm{irr}}}{\text{conjugacy}},$$

and it is known that $\lambda(M)$ is determined by the Dehn surgery formula(c.f. [1]). On the other hand, $\lambda(M)$ is the only one non-trivial invariant which is finite type of degree 1 for both the algebraically split link surgery and Torelli surgery.

Let X be an oriented compact smooth spin 4-manifold with boundary $\partial X = M$, such X always exists. Rohlin invariant $\mu(M)$ of M is defined by

$$\mu(M) \equiv \frac{\operatorname{Sign} X}{8} \pmod 2,$$

and there is a formula

$$\lambda(M) \equiv \mu(M) \pmod 2.$$

Namely, $\lambda(M)$ is an integral lift of Rohlin invariant $\mu(M)$.

On the face of things, the definitions of Casson invariant and Rohlin invariant looks very different: one is come from the flat $SU(2)$ connections, and the another one is from 4-dimensional. But some relations between Casson invariant and the signature of 4-manifolds are known.

Key words and phrases. Casson invariant, signature, configuration space, framing, finite type invariant.

Here are such two examples as follows. Let $V(a_1, \ldots, a_n)$ be the Milnor fiber of the Seifert homology 3-sphere $\Sigma(a_1, \ldots, a_n)$, then

$$-\frac{\operatorname{Sign} V(a_1, \ldots, a_n)}{8} = \lambda(\Sigma(a_1, \ldots, a_n))$$

([7],[8]). The another example is about Ohta's invariant $\tau(M)$ (preprint). He constructed some oriented compact smooth 4-manifold N with boundary $\partial N \cong M$ in the moduli space of anti-self dual connections on some principal $SU(2)$ bundle $P \to M \times S^1$, and proved that N is spin and

$$\tau(M) = -\frac{\operatorname{Sign} N}{8}$$

is an topological invariant of M. Hence, one can see that

$$\tau(M) \equiv \mu(M) \pmod 2.$$

It is unknown if $\tau(M) = \lambda(M)$.

As mentioned above, this report gives some topological construction of Casson invariant by using the configuration spaces of 3-manifolds. This results depends on the work by Kuperberg-Thurston [11] that relate our invariant to Casson invariant. Our construction corresponds to the first non-trivial term of their invariant. In [11], they gave a purely topological definition of the perturbative quantum invariants of links and 3-manifolds. Ordinally, this kind of work for the definition of the perturbative quantum invariants of 3-manifolds and links is by Kontsevich [10]. The related works, which uses the configuration spaces, was given by Axelrod-Singer [2, 3], Bott-Taubes [6], later by Bott-Cattaneo [4, 5], and Kuperberg-Thurston.

2. Configuration space of points and Gauss map

Two point configuration space $C_2(X)$ of a space X is defined by

$$C_2(X) = \{(x, y) \mid x, y \in X,\ x \neq y\}.$$

2.1. **Euclidean space.** Let

$$\varphi_{\mathbb{R}^3} \colon C_2(\mathbb{R}^3) \to S^2$$

be the map defined by

$$\varphi_{\mathbb{R}^3}(x, y) = \frac{y - x}{\|y - x\|}$$

for $(x, y) \in C_2(\mathbb{R}^3)$ which is called Gauss map. There exists a homeomorphism

$$C_2(\mathbb{R}^3) \cong \mathbb{R}^3 \times (0, \infty) \times S^2, \quad (x, y) \mapsto (z, r, v)$$

by the corresponding

$$z = \frac{x+y}{2}, \quad r = \frac{\|y-x\|}{2}, \quad v = \varphi_{\mathbb{R}^3}(x,y).$$

Immediately, the pre-image ${\varphi_{\mathbb{R}^3}}^{-1}(v)$ of a point $v \in S^2$ is contractible. In particular, its signature is zero. As we will see later (Theorem 2), this vanishing corresponds to the fact $\lambda(S^3) = 0$.

2.2. **The map** $\varphi_f\colon \partial C_2(\hat{M}) \to S^2$**.** We compactify $C_2(\hat{M})$ by the method of Bott-Taubes's paper. In this subsection, we construct a "partial Gauss map" $\varphi_f\colon U \to S^2$, where U is a complement of a certain compact subset of $C_2(\hat{M})$ by using some additional data f. By the definition of the compactification of $C_2(\hat{M})$, φ_f defined on U naturally extends to $\partial C_2(\hat{M})$.

Let M be an oriented closed homology 3-sphere and set $\hat{M} = M \# \mathbb{R}^3$. Let $f\colon T\hat{M} \to \hat{M} \times \mathbb{R}^3$ be a framing of $T\hat{M}$. In this report, we always assume that any framing of $T\hat{M}$ is compatible with Euclidean framing $f_{\mathbb{R}^3}$ over the complement of some compact subset of $\hat{M}$. Now, we define a map $\varphi_f\colon U \to S^2$ as follows, where $U = U_1 \cup U_2 \cup U_3$. Each U_i is defined in later three cases.

2.2.1. *Two points are very close.* First, let

$$U_1 = \{(x,y) \in C_2(\hat{M}) \mid d(x,y) < \varepsilon\},$$

where d is a metric of $\hat{M}$ and $\varepsilon > 0$ is small enough. Suppose $(x,y) \in U_1$. Then we can define the "direction" $\varphi_f(x,y) \in S^2$ from x to y by using f.

2.2.2. *one point is in the end of* $\hat{M}$. Suppose that $\hat{M}$ is obtained by connecting a 3-disk $D_p \subset M$ around $p \in M$ and $D^3 \subset \mathbb{R}^3$ around 0. Let $V \subset \hat{M}$ be the open subset corresponding to $M \setminus D_p$, and $W \subset \hat{M}$ corresponding $\mathbb{R}^3 \setminus D^3$. Let

$$U_2 = (V \times W) \cup (W \cup V).$$

We assume that f is coincides with $f_{\mathbb{R}^3}$ on some open set including the closure of W.

If $x \in V$ and $y \in W$, then define $\varphi_f(x,y) = y$. Note that this definition makes sense, because we can think y as a point in $\mathbb{R}^3$. Also define $\varphi_f(y,x) = -y$.

2.2.3. *Both two points in the end.* Let

$$U_3 = C_2(W),$$

and suppose $(x, y) \in U_3$. In this case, we define

$$\varphi_f(x, y) = \frac{y - x}{\|y - x\|}.$$

2.3. **TEST.** What we see in this subsection is how we get an invariant of (M, f) which is an integral lift of Rohlin invariant under the assumption of the existence of a map

$$\tilde{\varphi}_f \colon C_2(\hat{M}) \to S^2$$

such that $\tilde{\varphi}_f|_{\partial C_2(\hat{M})} = \varphi_f$.

Suppose (M, f) satisfies the above assumption. Taking a regular value $v \in S^2$ of $\tilde{\varphi}_f$, set

$$X_f = \tilde{\varphi}_f^{-1}(v).$$

Then X_f is an oriented compact smooth 4-manifold satisfying

(1) $\partial X_f \cong M \# M \# (-M)$,
(2) X_f is spin,
(3) the signature σ_f of X_f depends only on (M, f).

The property (1) is from the definition of φ_f. Since the normal bundle of $X_f \subset C_2(\hat{M})$ and the tangent bundle $TC_2(\hat{M})$ are spin, X_f is also spin, so we have (2). Another manifold $X_f{}'$ come from another map $\tilde{\varphi}'_f \colon C_2(\hat{M}) \to S^2$ is cobordant to X_f relative to the boundary, hence. Consequently, σ_f is an invariant of a pair (M, f), and we obtain (3). By the definition of Rohlin invariant, one can see

$$\frac{\sigma_f}{8} \equiv \mu(M) \pmod 2.$$

Therefore, we get an integral lift $\sigma_f/8$ of Rohlin invariant.

3. Two numbers d_f and σ_f

In this section, we define two integers d_f and σ_f used to define the invariant $I(M)$.

Let

$$\bar{C}_2(\hat{M}, f) = S^2 \cup_{\varphi_f} C_2(\hat{M})$$

be the attaching space by the map $\varphi_f \colon \partial C_2(\hat{M}) \to S^2$. Using the long exact sequence of $(\bar{C}_2(\hat{M}, f), S^2)$, we have an isomorphism

$$H^k(\bar{C}_2(\hat{M}, f); \mathbb{Z}) \cong \begin{cases} \mathbb{Z}, & \text{if } k = 0, 2, 4, 6 \\ 0, & \text{otherwise.} \end{cases}$$

The following lemma define the integer d_f, this definition is similar to the cohomological definition of the Hopf invariant(c.f. [9]).

Lemma 3.1. *There exists a graded ring isomorphism*

$$H^*(\bar{C}_2(\hat{M}, f); \mathbb{Z}) \cong \mathbb{Z}[a, b]/(a^2 - d_f b, b^2)$$

for some integer d_f*, where* $\deg a = 2$*,* $\deg b = 4$*.*

This number d_f is nothing but the Casson invariant for the framed 3-manifold ([11]).

Let $L_{S^2} \to S^2$ be a complex line bundle with Euler number 1, and s_{S^2} a generic section. We can assume that ${s_{S^2}}^{-1}(0)$ consists just one point. Set $L_f = {\varphi_f}^* L_{S^2}$, $s_f = {\varphi_f}^* s_{S^2}$. Then $s_f : \partial C_2(\hat{M}) \to L_f$ is a generic section, and ${s_f}^{-1}(0) \cong M \# M \# (-M)$. Since the inclusion $\partial C_2(\hat{M}) \hookrightarrow C_2(\hat{M})$ induces an isomorphism on H^2, there exists only one isomorphism class of a complex line bundle $\tilde{L}_f \to C_2(\hat{M})$ such that $\tilde{L}_f|_{\partial C_2(\hat{M})} = L_f$.

$$\begin{array}{ccc} L_f & \longrightarrow & \tilde{L}_f \\ \big\downarrow & & \big\downarrow \\ \partial C_2(\hat{M}) & \longrightarrow & C_2(\hat{M}) \end{array}$$

Let $\tilde{s}_f : C_2(\hat{M}) \to \tilde{L}_f$ be a generic section such that $\tilde{s}_f|_{\partial C_2(\hat{M})} = s_f$. Let

$$X_f = \tilde{s}_f^{-1}(0),$$

then X_f is an oriented compact smooth 4-manifold with boundary $\partial X_f \cong M \# M \# (-M)$. Define

$$\sigma_f = \operatorname{Sign} X_f.$$

Another choices $\tilde{s}'_f$ give the save value of σ_f, because a generic homotopy between $\tilde{s}_f$ and $\tilde{s}'_f$ gives a cobordism between X_f and X'_f.

Definition 3.2.

$$I(M) = \frac{d_f - \sigma_f}{8}$$
$$I(M, f) = (I(M), d_f)$$

4. Statements and Examples

Let (M, f) be an oriented closed homology 3-sphere with a framing of $T\hat{M}$. Let $I(M)$ be the number defined in Definition 3.2.

Theorem 1 ([14])**.** $I(M)$ *is a* $\mathbb{Z}$*-valued topological invariant of* M*, and it equals to Casson invariant of* M*.*

Now, we see some examples of the calculations of $I(M)$ and $I(M, f)$.

4.1. **Case of $M = S^3$ with Euclidean framing.** Let $M = S^3$, then $\hat{M} = \mathbb{R}^3$. Let $f_{\mathbb{R}^3}$ be the Euclidean framing on $T\mathbb{R}^3$, and we have $\varphi_{f_{\mathbb{R}^3}} = \varphi_{\mathbb{R}^3}$.

One can take the line bundle $\tilde{L}_{f_{\mathbb{R}^3}} \to C_2(\hat{M})$ as the pull-back bundle ${\varphi_{f_{\mathbb{R}^3}}}^* L_{S^2}$, and the pull-back section

$$\tilde{s}_{f_{\mathbb{R}^3}} = \varphi^*_{\mathbb{R}^3} s_{S^2} \colon C_2(\hat{M}) \to \tilde{L}_{f_{\mathbb{R}^3}}$$

is generic. If $v = s_{S^2}^{-1}(0)$, then we have

$$X_{f_{\mathbb{R}^3}} = \tilde{s}_{f_{\mathbb{R}^3}}^{-1}(0) = \varphi_{\mathbb{R}^3}^{-1}(v) \cong \mathbb{R}^4,$$

and so, we obtain $\sigma_{f_{\mathbb{R}^3}} = 0$.

Since $\bar{C}_2(\mathbb{R}^3) \cong S^4 \times S^2$, we have the ring isomorphism

$$H^*(\bar{C}_2(\hat{M}, f); \mathbb{Z}) \cong R_0,$$

this implies $d_{f_{\mathbb{R}^3}} = 0$. Therefore, we have

$$I(S^3) = \frac{d_{f_{\mathbb{R}^3}} - \sigma_{f_{\mathbb{R}^3}}}{8} = \frac{0-0}{8} = 0$$

and

$$I(S^3, f_{\mathbb{R}^3}) = (I(M), d_{f_{\mathbb{R}^3}}) = (0, 0).$$

4.2. **Case of $M = S^3$ with any framings.** Let f be an any framing of $\mathbb{R}^3$ which is always obtained by $f = g f_{\mathbb{R}^3}$ for some compact supported map $g \colon \mathbb{R}^3 \to SO(3)$. Any framings on $\mathbb{R}^3$ are classified by the degree $\deg g \in \mathbb{Z}$ of the induced homomorphism $g^* \colon H^3_c(SO(3); \mathbb{Z}) \to H^3_c(\mathbb{R}^3; \mathbb{Z})$.

Let $n = \deg g$. The map $\varphi_f \colon \partial C_2(\hat{M}) \cong S^3 \times S^2 \to S^2$ essentially equals to the evaluation map ev_g as follows:

$$ev_g \colon S^3 \times S^2 \to S^2, \quad (x, v) \mapsto g(x)v$$

Therefore, we have $\bar{C}_2(\hat{M}, f) \cong S(E_n)$, where $\pi \colon E_n \to S^4$ is a real vector bundle with $\langle [S^4], p_1(E_n) \rangle = 4n$ and $S(E_n)$ the associated sphere bundle. Calculating the characteristic classes of $TS(E_n)$ and $\pi^* E_n$([13]), we obtain that $d_f = n$ and $\sigma_f = n$. This implies that

$$I(S^3, f) = (\frac{d_f - \sigma_f}{8}, d_f) = (0, n),$$

and of course, we obtain $I(S^3) = 0$ again.

4.3. **Connected sum.** Let (M_1, f_1), (M_2, f_2) be framed manifolds. Set $M = M_1 \# M_2$, $f = f_1 \# f_2$. Let us think $\hat{M}_1 = M_1 \# \mathbb{R}_1$, $\hat{M}_2 = M_2 \# \mathbb{R}_2$, where

$$\mathbb{R}_1 = \{(x_1, x_2, x_3) \mid x_1 < 0\}, \quad \mathbb{R}_2 = \{(x_1, x_2, x_3) \mid x_1 > 0\},$$

and each f_i is a faming over $\hat{M}_1$ compatible with the Euclidean framing on the end. Moreover, we suppose that $\hat{M}_i$ is the connected sum at around a point in M_i and around $(\pm R, 0, 0) \in \mathbb{R}_i$ for some large number $R \gg 1$. Then we can take $\hat{M}$ as

$$\hat{M}_1 \cup V \cup \hat{M}_2$$

such that the M_1-part and M_2-part in $\hat{M}$ are very far each other, where $V = (-1, 1) \times \mathbb{R}^2$.

Next, we will construct $\varphi_f \colon \partial C_2(\hat{M}) \to S^2$. Define a map

$$\varphi_{ij} \colon (V \cup \hat{M}_i) \times (V \cup \hat{M}_j) \setminus \Delta \to S^2$$

as follows $(i, j = 1, 2,\ i \neq j)$. Let $h \colon \hat{M} \to \mathbb{R}^3$ be the map obtained by collapsing each M_i-part to $(\pm R, 0, 0)$. Then $\varphi_{ij}(x, y)$ is defined by

$$\varphi_{ij}(x, y) = \frac{h(y) - h(x)}{\|h(y) - h(x)\|}.$$

Let $\tilde{\varphi}_{f_i} \colon C_2(M_i) \to \mathbb{C}P^3$ be the classifying map of $\tilde{L}_{f_i}$. Let $\tilde{\varphi}_f \colon C_2(\hat{M}) \to \mathbb{C}P^3$ be an extension map of $\tilde{\varphi}_f$ obtained from $\tilde{\varphi}_{f_1}$, $\tilde{\varphi}_{f_2}$ and φ_{ij}. Note that any two such maps are coincide on these common domain. Let $\mathbb{C}P^{2'} \subset \mathbb{C}P^3$ be a submanifold homologous to $\mathbb{C}P^2$ that transversally intersect with $\mathbb{C}P^1 \cong S^2$ at one point $v = (1, 0, 0) \in S^2$. There exists a generic section of the complex line bundle $L_{\mathbb{C}P^3} \to \mathbb{C}P^3$ with $c_1(L_{\mathbb{C}P^3}) = 1$ such that the pre-image of zero is $\mathbb{C}P^{2'}$. Hence

$$X_f = \varphi_f(\mathbb{C}P^{2'}) \cong X_{f_1} \sharp X_{f_2},$$

this means $\sigma_f = \sigma_{f_1} + \sigma_{f_2}$. And also it is easy to see that $d_f = d_{f_1} + d_{f_2}$. Therefore, we obtain the following:

Proposition 4.1.

$$I(M, f) = I(M_1, f_1) + I(M_2, f_2)$$

$$I(M) = I(M_1) + I(M_2)$$

4.4. **Opposite orientation.** Let M' be M with the opposite orientation, and let $f' = (-f_1, f_2, f_3)$ where $f = (f_1, f_2, f_3)$. By the definition of d_f and σ_f, we have

$$d_{f'} = -d_f, \quad \sigma_{f'} = -\sigma_f.$$

This implies that

Proposition 4.2.

$$I(M', f') = -I(M, f)$$

$$I(M') = -I(M)$$

5. Outline of proof

5.1. **Integrality of $I(M)$.** Since $\Omega_5^{spin}(S^2) = 0$([15]), there exists an oriented compact smooth spin 6-manifold Z with a complex line bundle $L_Z \to Z$ such that

$$\partial Z = \partial C_2(\hat{M}), \quad L_Z|_{\partial C_2(\hat{M})} = L_f$$

and the image of the classifying map of L_Z is contained in S^2. Let

$$W = C_2(\hat{M}) \cup_{\partial C_2(\hat{M})} Z, \quad L_W = \tilde{L}_f \cup_{L_f} L_Z \to W.$$

Then, (W, L_W) is an oriented closed smooth spin 6-manifold with a complex line bundle. Applying the index theorem to (W, L_W), one can see that the integral

$$\int_W ch(L_W)\hat{\mathcal{A}}(TW)$$

is an integer(c.f. [12]). Here, ch is the Chern character and $\hat{\mathcal{A}}$ is the $\hat{\mathcal{A}}$-genus. This value equals to

$$I(M) - \frac{\operatorname{Sign} X_Z}{8},$$

where X_Z is the pre-image of 0 of a generic section, which is an extension of s_f of L_f, of L_Z. Since X_Z is spin and ∂X_Z is a homology 3-sphere, we have $\operatorname{Sign} X_Z \equiv 0 \pmod 8$. Therefore, we obtain the following proposition.

Proposition 5.1. *The number $I(M)$ is an integer.*

5.2. **Topological invariance of $I(M)$.** Let f, f' be framings of $T\hat{M}$. f' can be represented by $f' = gf$ for some $g\colon \hat{M} \to SO(3)$. Then $(M, f') \cong (M\#S^3, f\#gf_{\mathbb{R}^3})$. According to Proposition 4.1,

$$I(M, f') = I(M, f) + I(S^3, gf_{\mathbb{R}^3}) = I(M, f) + (0, \deg g).$$

In other words,

$$d_{f'} = d_f + \deg g, \quad \sigma_{f'} = \sigma_f + \deg g.$$

In particular, $I(M)$ does not depend on f. Therefore

Proposition 5.2. *$I(M)$ is an topological invariant of M.*

5.3. **Casson invariant.** Kuperberg-Thurston showed that some value $\tilde{I}_1(M)$ is Casson invariant by using the theory of finite type invariants of homology 3-spheres in their paper [11]. The invariant $\tilde{I}(M)$ is constructed as follows. First, define

$$I_1(M) = \frac{1}{6}\langle \bar{C}_2(\hat{M}, f), c_1(L_f)^3\rangle,$$
$$\delta_1(M) = \frac{1}{24}\langle X_f, p_1(TC_2(\hat{M})|_{X_f})\rangle.$$

And then $I(M)$ is defined by

$$\tilde{I}_1(M) = I_1(M) - \delta_1(M).$$

Calculating characteristic classes, we have the following proposition.

Proposition 5.3. $\tilde{I}_1(M) = I(M)$.

By Proposition 5.3, we obtain Theorem 1. To prove that $I(M)$ is a $\mathbb{Z}$-valued topological invariant, one only need Proposition 5.1,5.2.

6. Casson invariant as a signature

By § 5.2, we obtain the following proposition.

Proposition 6.1. *There exists only one framing f_M of $T\hat{M}$ such that $d_{f_M} = 0$.*

Therefore, we have

$$I(M) = \frac{d_{f_M} - \sigma_{f_M}}{8} = -\frac{\sigma_{f_M}}{8}.$$

Theorem 2.

$$\lambda(M) = -\frac{\sigma_{f_M}}{8}.$$

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