The knot $K_n^{(3,q)}$

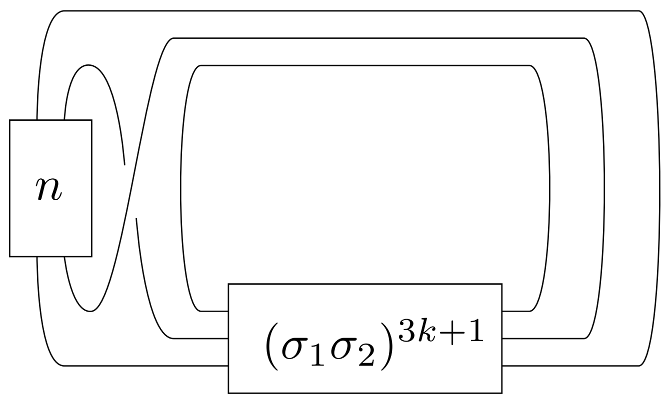

FIGURE 1. The knot $K_n^{(3,3k+1)}$. σ_1 and σ_2 are standard generators of the 3-braid group. The box with number n indicates n right handed vertical full-twists.

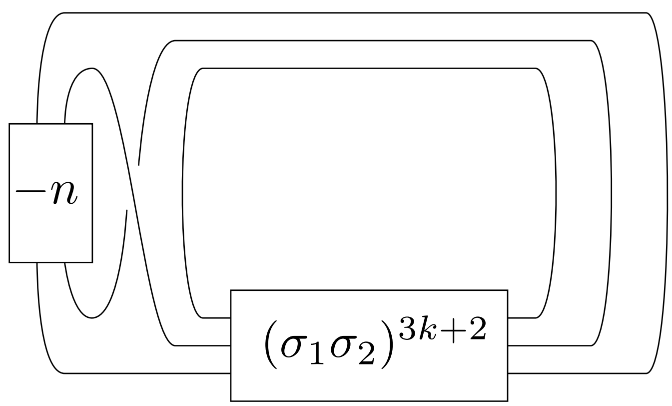

FIGURE 2. The knot $K_n^{(3,3k+2)}$. The box with number $-n$ indicates n left handed vertical full-twists.

$\underline{\mathrm{CFK}^\infty(K_n^{(3,q)})}$

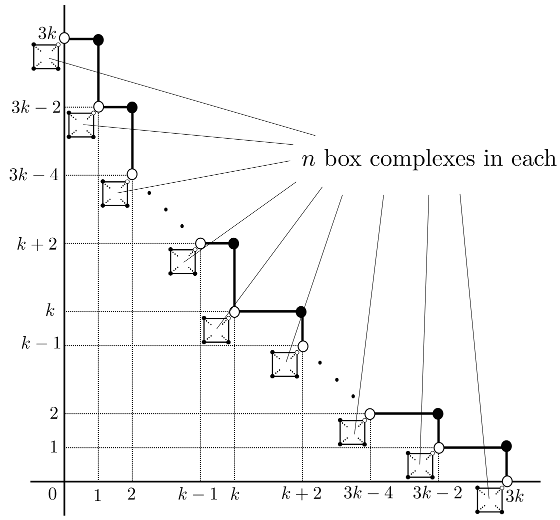

FIGURE 3. The full knot Floer complex $\mathrm{CFK}^\infty(K_n^{(3,3k+1)})$. This complex consists of the staircase complex, which is consistent with the complex of $\mathrm{CFK}^\infty(T(3,3k+1))$, and box complexes. The vertices of box complexes are actually on the grid, but are drawn slightly displaced.

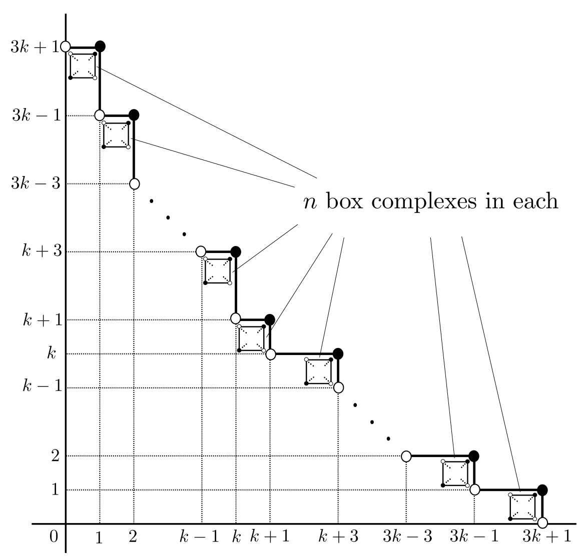

FIGURE 4. The full knot Floer complex $\mathrm{CFK}^\infty(K_n^{(3,3k+2)})$. The staircase complex is consistent with $\mathrm{CFK}^\infty(T(3,3k+2))$

$\widehat{\mathrm{HFK}}(K_n^{(3,q)})$ and the Alexander polynomial

$$\operatorname{rank}\widehat{\mathrm{HFK}}_d(K_n^{(3,3k+1)};i)=\begin{cases} n & (d,i)=(1-2j,3k+1-3j) & (j=0,\ldots,k)\\ n & (d,i)=(-2k-1-4j,-1-3j) & (j=0,\ldots,k)\\ 2n+1 & (d,i)=(-2j,3k-3j) & (j=0,\ldots,k)\\ 2n+1 & (d,i)=(-2k-4-4j,-3-3j) & (j=0,\ldots,k-1)\\ n+1 & (d,i)=(-1-2j,3k-1-3j) & (j=0,\ldots,k-1)\\ n+1 & (d,i)=(-2k-3-4j,-2-3j) & (j=0,\ldots,k-1)\\ 0 & \text{otherwise,} \end{cases}$$

$$\operatorname{rank}\widehat{\mathrm{HFK}}_d(K_n^{(3,3k+2)};i)=\begin{cases} n+1 & (d,i)=(-2j,3k+1-3j) & (j=0,\ldots,k)\\ n+1 & (d,i)=(-2k-2-4j,-1-3j) & (j=0,\ldots,k)\\ 2n+1 & (d,i)=(-1-2j,3k-3j) & (j=0,\ldots,k)\\ 2n+1 & (d,i)=(-2k-5-4j,-3-3j) & (j=0,\ldots,k-1)\\ n & (d,i)=(-2-2j,3k-1-3j) & (j=0,\ldots,k-1)\\ n & (d,i)=(-2k-4-4j,-2-3j) & (j=0,\ldots,k-1)\\ 0 & \text{otherwise.} \end{cases}$$

For a knot K, $\Delta_K(t)=\sum_{d,i}(-1)^d t^i\cdot\operatorname{rank}\widehat{\mathrm{HFK}}_d(K;i)$ [Ozváth–Szabó].

$$\begin{aligned}\Delta_{K_n^{(3,3k+1)}}(t)=&\sum_{i=1}^{k}\{-nt^{3i+1}+(2n+1)t^{3i}-(n+1)t^{3i-1}\}\\ &-nt+(2n+1)-nt^{-1}\\ &+\sum_{i=1}^{k}\{-(n+1)t^{-3i+1}+(2n+1)t^{-3i}-nt^{-3i-1}\}.\end{aligned}$$

$$\begin{aligned}\Delta_{K_n^{(3,3k+2)}}(t)=&\sum_{i=1}^{k}\{(n+1)t^{3i+1}-(2n+1)t^{3i}+nt^{3i-1}\}\\ &(n+1)t-(2n+1)+(n+1)t^{-1}\\ &+\sum_{i=1}^{k}\{nt^{-3i+1}-(2n+1)t^{-3i}+(n+1)t^{-3i-1}\}.\end{aligned}$$

A doubly pointed Heegaard diagram of $K_n^{(3,3k+1)}$ and $\mathrm{CFK}^\infty(K_1^{(3,4)})$

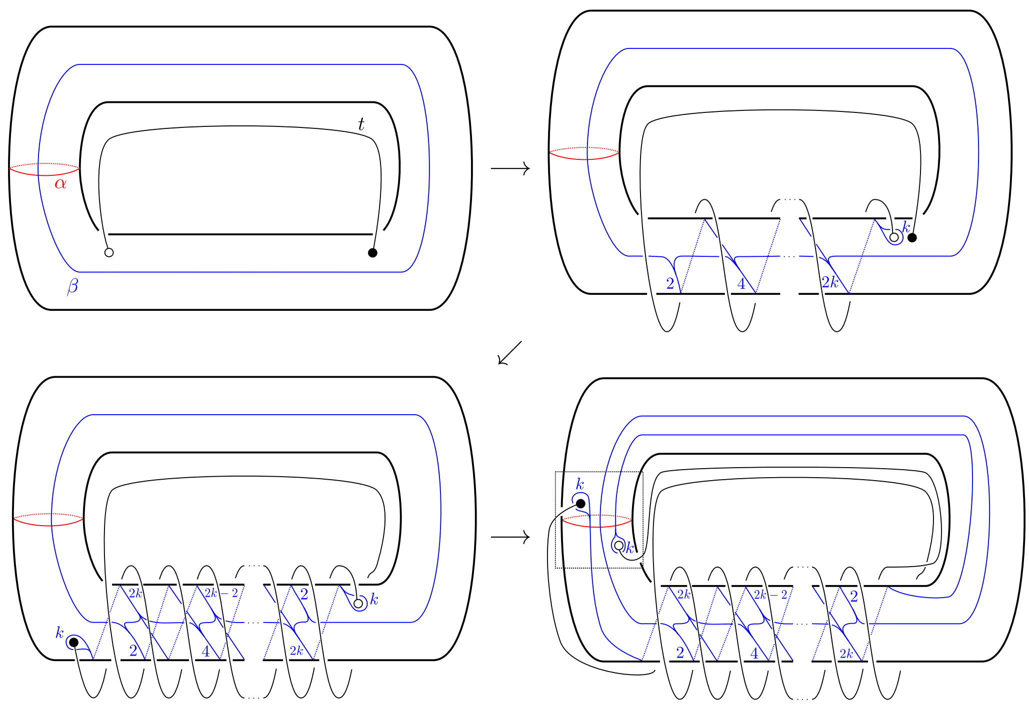

FIGURE 5. How to move the arc t to get a genus one doubly pointed Heegaard diagram of $K_n^{(3,3k+1)}$. The bottom right is the diagram of $K_0^{(3,3k+1)} = T_{3,3k+1}$. To obtain the diagram of $K_n^{(3,3k+1)}$, we need to add n-twists clockwisely in the region indicated the dotted box.

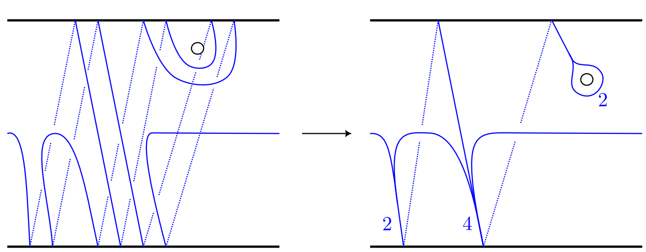

FIGURE 6. Parallel arcs are merged into one curve with multiplicity.

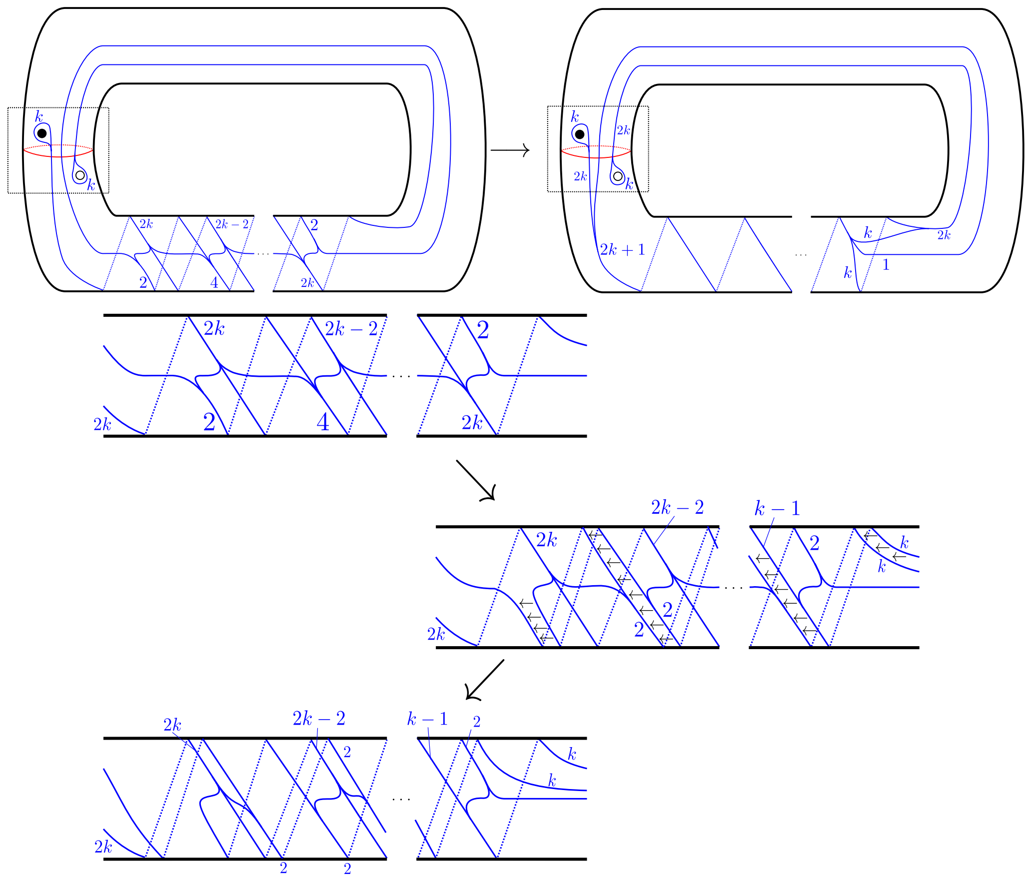

FIGURE 7. The β curve is neatly aligned. Bottom is a detailed description of how to aline the β curve.

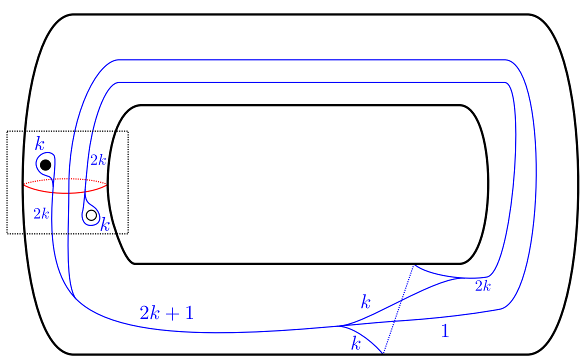

FIGURE 8. A diagram of $K_0^{(3,3k+1)}$ after left handed Dehn twists along the α curve.

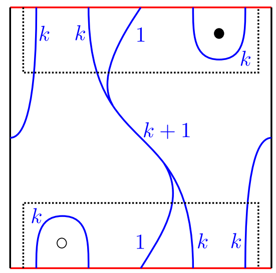

FIGURE 9. The rectangle is obtained by cutting the diagram in Figure 8 along α and the standard longitude. When it restores the torus, the top and bottom sides (red) become the α curve.

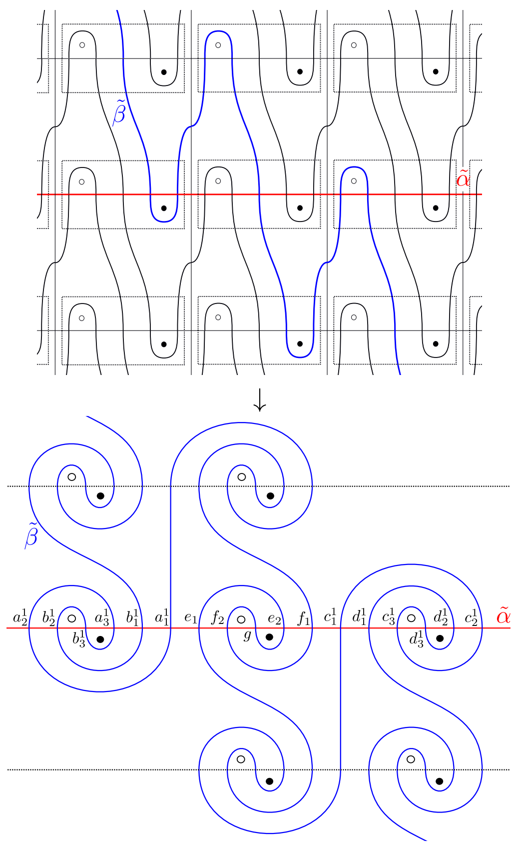

FIGURE 10. (Top) The universal cover of a diagram of $K_0^{(3,4)}$ (see Figure 9). Let $\tilde{\alpha}$ (red) and $\tilde{\beta}$ (blue) be one connected components of lifts of α and β, respectively. (Bottom) The universal cover of a diagram of $K_1^{(3,4)}$. It can be obtained from the top figure by twisting two points once clockwisely in the dotted box. The labeling of $\tilde{\alpha} \cap \tilde{\beta}$ is indicated to the left of each point. Note that the white dot is z and the black dot is w.

from	to	base point	from	to	base point
a_1^1	a_2^1	$\bullet$	c_1^1	c_2^1	$\circ$
	b_1^1	$\bullet$		d_1^1	$\circ$
	f_1	$\circ\circ$		f_1	$\bullet\bullet$
	e_1	$\circ\circ$		e_1	$\bullet\bullet$
a_2^1	a_3^1	$\circ$	c_2^1	c_3^1	$\bullet$
	b_2^1	$\bullet$		d_2^1	$\circ$
a_3^1	b_3^1	$\bullet$	c_3^1	d_3^1	$\circ$
b_1^1	a_3^1	$\circ$	d_1^1	c_3^1	$\bullet$
	b_2^1	$\bullet$		d_2^1	$\circ$
b_2^1	b_3^1	$\circ$	d_2^1	d_3^1	$\bullet$
f_1	f_2	$\bullet$	e_1	f_2	$\bullet$
	e_2	$\circ$		e_2	$\circ$
f_2	g	$\circ$	e_2	g	$\bullet$

TABLE 1. The list of disks contributing to differentials of $\mathrm{CFK}^\infty(K_1^{(3,4)})$. For example, there is a disk connecting a_1^1 to a_2^1 with one black dot w.

Alexander grading	generator
4	b_2^1
3	$a_2^1,\ b_1^1,\ b_3^1$
2	$a_1^1,\ a_3^1$
1	f_2
0	$g,\ f_1,\ e_1$
-1	e_2
-2	$c_1^1,\ c_3^1$
-3	$c_2^1,\ d_1^1,\ d_3^1$
-4	d_2^1

TABLE 2. The Alexander gradings of the generators of $\mathrm{CFK}^\infty(K_1^{(3,4)})$.

Change of basis:

- $b_1^1 \longrightarrow b_1^1 + a_2^1 =: B_1^1$,
- $d_1^1 \longrightarrow d_1^1 + c_2^1 =: D_1^1$,
- $f_1 \longrightarrow f_1 + e_1 =: F_1$.

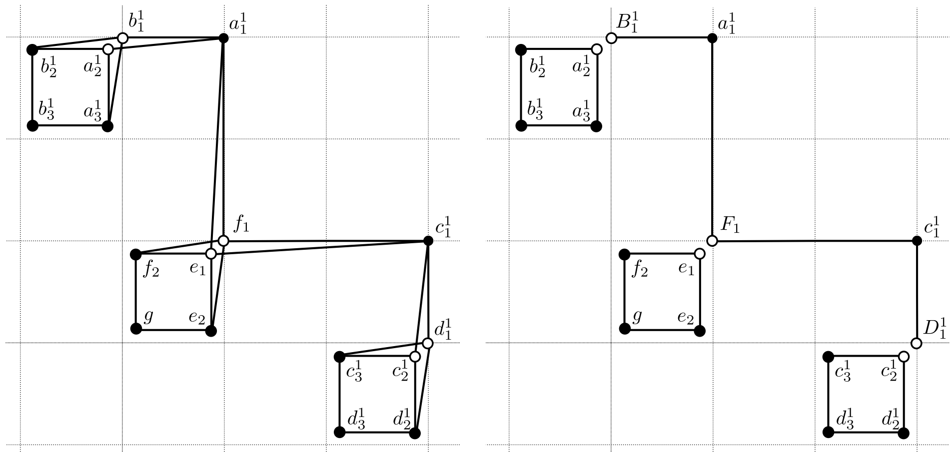

FIGURE 11. The full knot Floer complex $\mathrm{CFK}^\infty(K_1^{(3,4)})$. The information of the coefficient U is omitted. Left is the complex derived from Figure 10. By applying the change of basis, right is obtained.