

spatial graph とその invariant について, II

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Abstract

spatial graph, 特に trivalent vertex を持つ θ -curve や handcuff graph を分類する上では, その invariant が重要となります. invariant は前回紹介しましたので, 今回は 7 交点までの θ -curve と handcuff graph の分類を行ないます.

1. Yamada polynomial

2. Prime basic 4-regular disk graphs

3. Prime basic θ -polyhedra

4. Enumeration of θ -curve

以上の章は申し訳ありませんが, 前回のレジュメを御覧下さい.

5. Enumeration of handcuff graph

We give an enumeration of handcuff graph with up to seven crossings by using our notation. Links in the second column correspond to Rolfsen's knot table [15], and handcuff graphs in the last column correspond to Fig. 1-4. \bar{L} and $\bar{\Phi}$ denote mirror images of L and Φ , respectively.

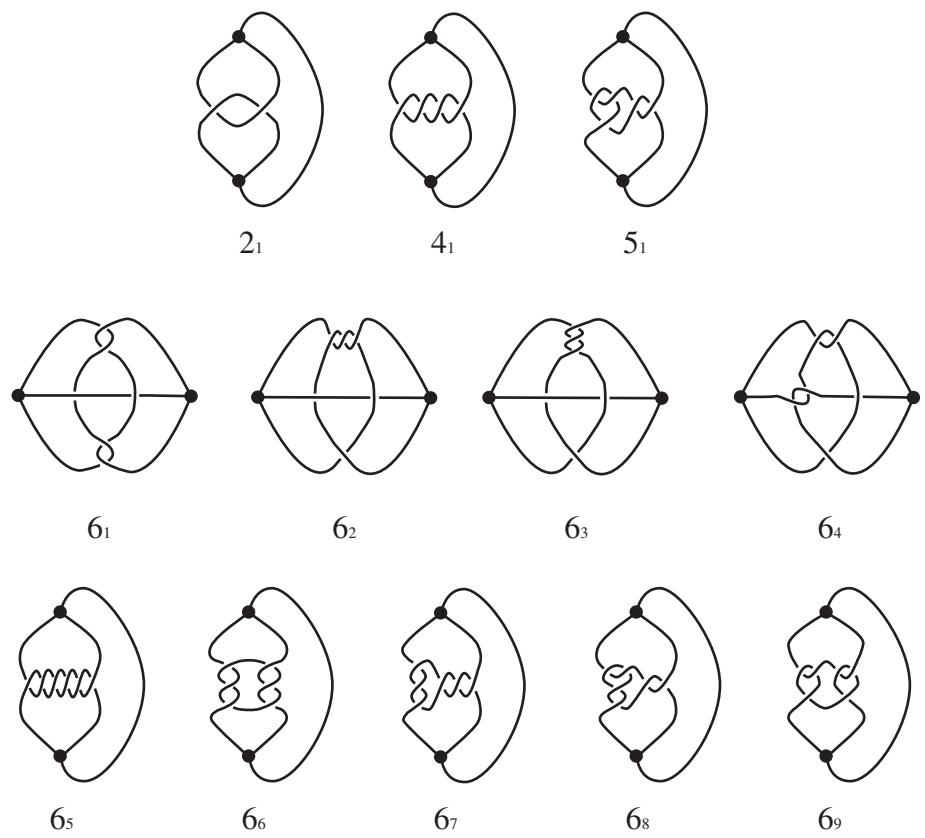


Fig. 1. Prime handcuff graphs with up to six crossings.

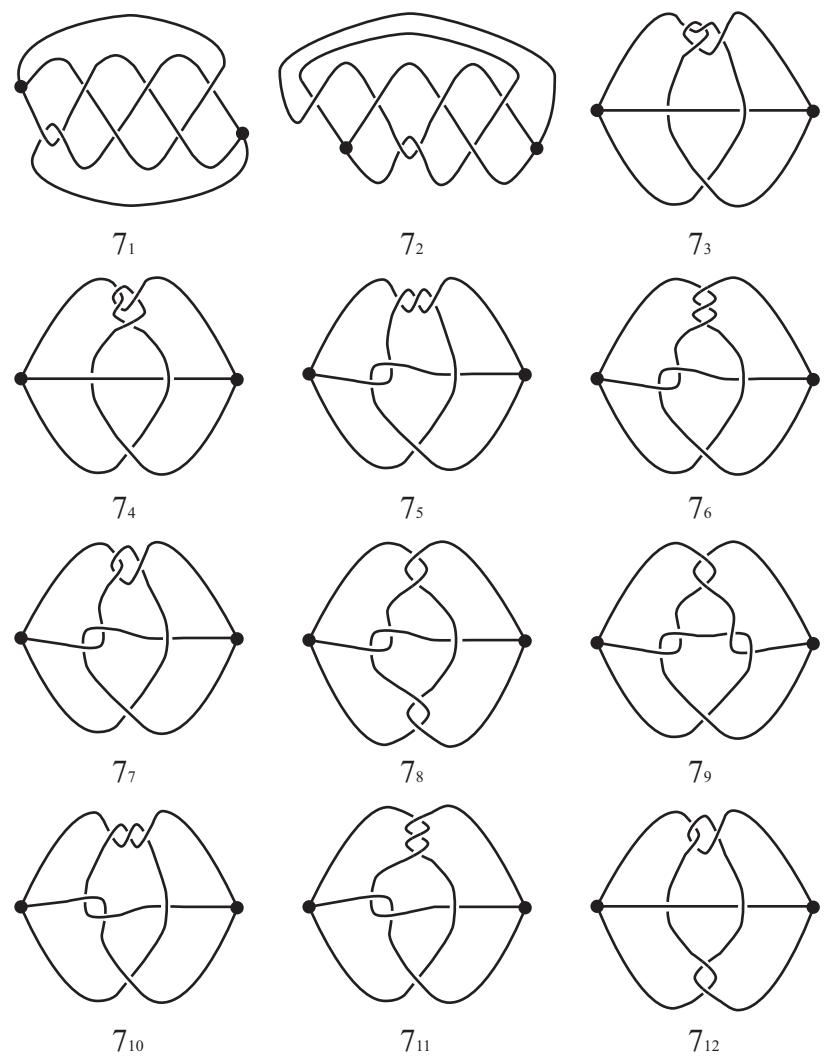


Fig. 2. Prime handcuff graphs with seven crossings.

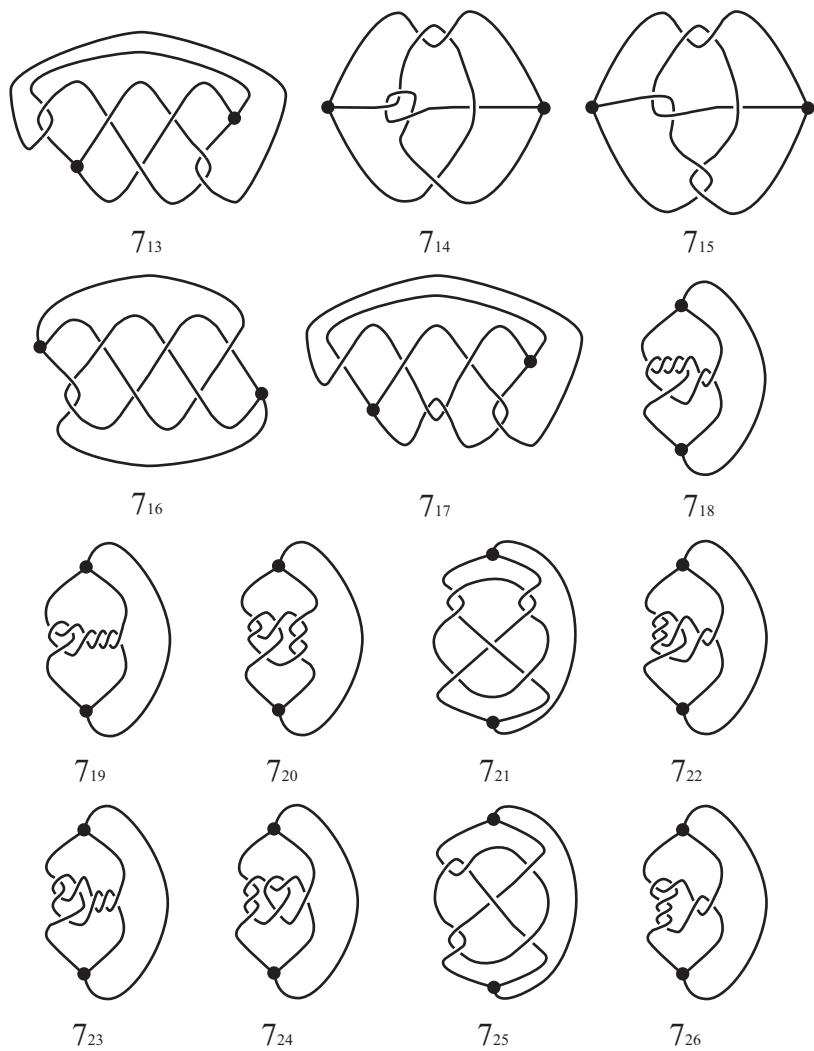


Fig. 3. Prime handcuff graphs with seven crossings (continued).

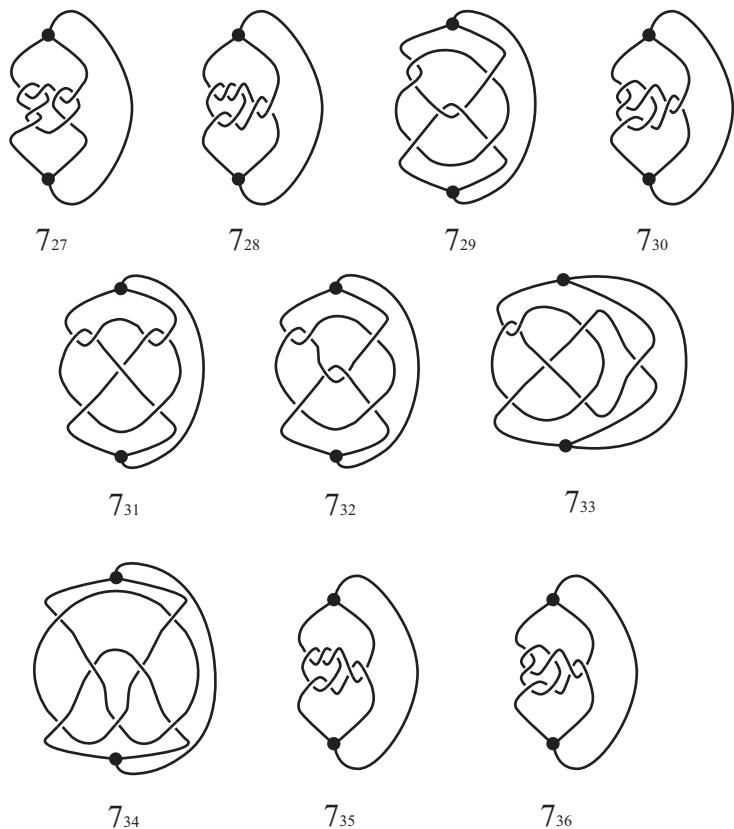


Fig. 4. Prime handcuff graphs with seven crossings (continued).

notation	constituent link	handcuff graph
$1_{\times}^1 2$	2_1^2	2_1^{ϕ}
$1_{\times}^1 4$	4_1^2	4_1^{ϕ}
$1_{\times}^1 2 \ 1 \ 2$	5_1^2	5_1^{ϕ}
$1_{\times}^1 6$	6_1^2	6_5^{ϕ}
$1_{\times}^1 3 \ 3$	6_2^2	6_7^{ϕ}
$1_{\times}^1 2 \ 2 \ 2$	6_3^2	6_8^{ϕ}
$1_{\times}^1 3, 3$	$\underline{6_1^2}$	6_6^{ϕ}
$1_{\times}^1 2 \ 1, 2 \ 1$	6_3^2	6_9^{ϕ}
$3_*^1 2 \ 0. - 2. 1$	$\underline{4_1^2}$	4_1^{ϕ}
$3_*^1 2 \ 0.2. - 1$	0_1^2	$2_1^{\phi} \#_3 2_1^{\phi}$
$3_*^1 - 2 \ 0. - 2 \ 0. 2$	2_1^2	$\underline{6_2^{\phi}}$
$3_*^1 2 \ 1 \ 0.2 \ 0. - 1$	4_1^2	6_4^{ϕ}
$4_*^1 3.1.1.1$	$\underline{2_1^2}$	6_2^{ϕ}
$4_*^1 3 \ 0.1.1.1$	$\underline{2_1^2}$	$\underline{6_3^{\phi}}$
$4_*^1 - 3 \ 0.1.1.1$	4_1^2	6_4^{ϕ}
$4_*^1 2.2.1.1$	$\underline{4_1^2}$	6_4^{ϕ}
$4_*^1 2 \ 0.2.1.1$	$\underline{2_1^2}$	6_3^{ϕ}
$4_*^1 - 2 \ 0.2.1.1$	$\underline{2_1^2}$	6_2^{ϕ}
$4_*^1 2 \ 0.1.1.2 \ 0$	0_1^2	6_1^{ϕ}
$4_*^1 2 \ 0.1.1. - 2 \ 0$	4_1^2	6_4^{ϕ}
$6_*^1 1.1.1.1.1.1$	0_1^2	6_1^{ϕ}
$6_*^3 1.1.1.1.1.1$	5_1^2	5_1^{ϕ}
$6_*^3 1.1.1.1. - 1. - 1$	2_1^2	$2_1^{\phi} \#_3 3_1^{\theta}$
$6_*^3 1.1. - 1.1.1.1$	2_1^2	$2_1^{\phi} \#_3 3_1^{\theta}$

notation	constituent link	handcuff graph
$1_{\times}^1 4 1 2$	$\overline{7_1^2}$	$\overline{7_{18}^\phi}$
$1_{\times}^1 2 1 4$	$\overline{7_1^2}$	$\overline{7_{19}^\phi}$
$1_{\times}^1 2 3 2$	$\overline{7_3^2}$	$\overline{7_{26}^\phi}$
$1_{\times}^1 3 1 1 2$	$\overline{7_2^2}$	$\overline{7_{22}^\phi}$
$1_{\times}^1 2 1 1 3$	$\overline{7_2^2}$	$\overline{7_{23}^\phi}$
$1_{\times}^1 2 1 1, 3$	$\overline{7_1^2}$	$\overline{7_{20}^\phi}$
$1_{\times}^1 2 2, 2 1$	$\overline{7_3^2}$	$\overline{7_{21}^\phi}$
$1_{\times}^1 3, 2 1 +$	$\overline{7_2^2}$	$\overline{7_{24}^\phi}$
$1_{\times}^1 3, -2 - 1 -$	$\overline{6_3^2}$	$\overline{6_9^\phi}$
$1_{\times}^1 (3, 2) 1 1$	$\overline{7_4^2}$	$\overline{7_{28}^\phi}$
$1_{\times}^1 (3, -2) 1 1$	$\overline{7_7^2}$	$\overline{7_{35}^\phi}$
$1_{\times}^1 (2 1, 2) 1 1$	$\overline{7_5^2}$	$\overline{7_{30}^\phi}$
$1_{\times}^1 (2 1, -2) 1 1$	$\overline{7_8^2}$	$\overline{7_{36}^\phi}$
$1_{\times}^1 (3, 2) 2$	$\overline{7_4^2}$	$\overline{7_{28}^\phi}$
$1_{\times}^1 (3, 2) - 2$	$\overline{7_7^2}$	$\overline{7_{35}^\phi}$
$1_{\times}^1 (3, -2) - 2$	$\overline{7_8^2}$	$\overline{7_{36}^\phi}$
$1_{\times}^1 (2 1, 2) 2$	$\overline{7_5^2}$	$\overline{7_{30}^\phi}$
$1_{\times}^1 (2 1, 2) - 2$	$\overline{7_8^2}$	$\overline{7_{36}^\phi}$
$1_{\times}^1 (2 1, -2) - 2$	$\overline{7_7^2}$	$\overline{7_7^\phi}$

notation	constituent link	handcuff graph
$3^1_* 2 2.2. - 1$	$\overline{5}_1^2$	7_{14}^ϕ
$3^1_* 2 1.2.2$	2_1^2	6_3^ϕ
$3^1_* 2 1.2. - 2$	2_1^2	7_7^ϕ
$3^1_* 2 1. - 2.2$	2_1^2	7_7^ϕ
$3^1_* 2 1. - 2. - 2$	2_1^2	7_3^ϕ
$3^1_* 3. - 2 0.2$	4_1^2	6_4^ϕ
$3^1_* 3 0.2 0. - 2$	$\overline{4}_1^2$	7_{10}^ϕ
$3^1_* 3 0. - 2 0.2$	4_1^2	6_4^ϕ
$3^1_* 2 1 0.2 0. - 2$	2_1^2	7_5^ϕ
$4^1_* 2 1 1.1.1.1$	2_1^2	7_3^ϕ
$4^1_* 2 1 1 0.1.1.1$	2_1^2	7_4^ϕ
$4^1_* - 2 - 1 - 1 0.1.1.1$	5_1^2	7_{14}^ϕ
$4^1_* 3.2 0.1.1$	4_1^2	7_{10}^ϕ
$4^1_* 3 0.2 0.1.1$	$\overline{4}_1^2$	7_{11}^ϕ
$4^1_* - 3 0.2 0.1.1$	2_1^2	7_7^ϕ
$4^1_* 2 1.2.1.1$	2_1^2	7_7^ϕ
$4^1_* 2 1 0.2.1.1$	$\overline{4}_1^2$	7_{11}^ϕ
$4^1_* - 2 - 1 0.2.1.1$	$\overline{4}_1^2$	7_{10}^ϕ
$4^1_* 3.2.1.1$	2_1^2	7_5^ϕ
$4^1_* 3 0.2.1.1$	2_1^2	7_6^ϕ
$4^1_* - 3 0.2.1.1$	2_1^2	7_5^ϕ
$4^1_* 2 1.1.1.20$	4_1^2	7_{12}^ϕ
$4^1_* 2 1.1.1. - 20$	$\overline{5}_1^2$	7_{14}^ϕ
$4^1_* 2.2 1 0.1.1$	5_1^2	7_{14}^ϕ
$4^1_* 2 0.2 1 0.1.1$	2_1^2	7_4^ϕ
$4^1_* - 2 0.2 1 0.1.1$	2_1^2	7_3^ϕ
$4^1_* 2 0.2.2.1$	4_1^2	7_{11}^ϕ
$4^1_* - 2 0.2.2.1$	0_1^2	7_2^ϕ
$4^1_* 2 0.2.2 0.1$	2_1^2	7_6^ϕ
$4^1_* - 2 0.2.2 0.1$	2_1^2	7_9^ϕ
$4^1_* 2.2 0.1.2 0$	5_1^2	7_{15}^ϕ
$4^1_* 2.2 0.1. - 2 0$	4_1^2	7_{10}^ϕ
$4^1_* 2 0.2.1.2 0$	2_1^2	7_8^ϕ
$4^1_* 2 0.2.1. - 2 0$	2_1^2	7_7^ϕ
$4^1_* - 2 0.2.1.2 0$	2_1^2	7_5^ϕ
$4^1_* - 2 0.2.1. - 2 0$	2_1^2	$2_1^\phi \#_3 \overline{3}_1^\theta$

notation	constituent link	handcuff graph
$5_{\times}^1 2.2.1.1.1$	$\overline{7_5^2}$	$\overline{7_{31}^\phi}$
$5_{\times}^1 2. - 2.1.1.1$	$\overline{7_8^2}$	$\overline{7_{36}^\phi}$
$5_{\times}^1 - 2. - 2.1.1.1$	$\overline{7_7^2}$	$\overline{7_{35}^\phi}$
$5_{\times}^1 2.1.2.1.1$	$\overline{7_5^2}$	$\overline{7_{32}^\phi}$
$5_{\times}^1 2.1. - 2.1.1$	$\overline{7_8^2}$	$\overline{7_{36}^\phi}$
$5_{\times}^1 - 2.1.2.1.1$	$\overline{6_3^2}$	$\overline{6_9^\phi}$
$5_{\times}^1 - 2.1. - 2.1.1$	$\overline{5_1^2}$	$\overline{5_1^\phi}$
$5_{\times}^1 2.1.1.2 0.1$	$\overline{7_2^2}$	$\overline{7_{25}^\phi}$
$5_{\times}^1 - 2.1.1.2 0.1$	$\overline{6_2^2}$	$\overline{7_{17}^\phi}$
$5_{\times}^1 2 0.1.2.1.1$	$\overline{7_4^2}$	$\overline{7_{29}^\phi}$
$5_{\times}^1 2 0.1. - 2.1.1$	$\overline{7_7^2}$	$\overline{7_{35}^\phi}$
$5_{\times}^1 2 0.2 0.1.1.1$	$\overline{7_1^2}$	$\overline{7_{21}^\phi}$
$5_{\times}^1 - 2 0. - 2 0.1.1.1$	$\overline{6_2^2}$	$\overline{7_{17}^\phi}$
$5_*^1 2 1 0.1.1.1.1$	2_1^2	$\overline{7_7^\phi}$
$5_*^1 - 2 - 1 0.1.1.1.1$	2_1^2	$\overline{7_7^\phi}$
$5_*^1 2 0.2 0.1.1.1$	0_1^2	$\overline{7_2^\phi}$
$5_*^1 - 2 0.2 0.1.1.1$	4_1^2	$\overline{7_{13}^\phi}$
$5_*^1 2 0.1.2 0.1.1$	2_1^2	$\overline{7_7^\phi}$
$5_*^1 - 2 0.1.2 0.1.1$	2_1^2	$\overline{7_3^\phi}$
$5_*^1 2 0.1. - 2 0.1.1$	2_1^2	$2_1^\phi \#_3 \overline{3_1^\theta}$
$5_*^1 - 2 0.1. - 2 0.1.1$	2_1^2	$\overline{6_2^\phi}$
$5_*^1 2 0.1.1.2 0.1$	2_1^2	$\overline{7_3^\phi}$
$5_*^1 - 2 0.1.1.2 0.1$	2_1^2	$\overline{7_7^\phi}$
$5_*^1 2 0.1.1. - 2 0.1$	2_1^2	$\overline{6_2^\phi}$
$5_*^1 - 2 0.1.1. - 2 0.1$	2_1^2	$2_1^\phi \#_3 \overline{3_1^\theta}$
$5_*^1 2 0.1.1.1.2 0$	4_1^2	$\overline{7_{13}^\phi}$
$5_*^1 - 2 0.1.1.1.2 0$	0_1^2	$\overline{7_2^\phi}$
$5_*^1 - 1.2 0.2 0.1.1$	6_3^2	$\overline{6_9^\phi}$
$5_*^1 - 1. - 2 0.2 0.1.1$	$\overline{4_1^2}$	$\overline{6_4^\phi}$
$5_*^1 - 1.2 0. - 2 0.1.1$	5_1^2	$\overline{5_1^\phi}$
$5_*^1 - 1. - 2 0. - 2 0.1.1$	$\overline{5_1^2}$	$\overline{5_1^\phi}$
$5_*^1 1.1.1.2 0.2 0$	$\overline{6_3^2}$	$\overline{6_9^\phi}$
$5_*^1 1.1.1. - 2 0.2 0$	$\overline{5_1^2}$	$\overline{5_1^\phi}$
$5_*^1 1.1.1.2 0. - 2 0$	4_1^2	$\overline{6_4^\phi}$
$5_*^1 1.1.1. - 2 0. - 2 0$	5_1^2	$\overline{5_1^\phi}$
$5_*^1 1.1.2.1.2 0$	6_2^2	$\overline{7_{17}^\phi}$
$5_*^1 - 1.2 0.1.2.1$	6_2^2	$\overline{7_{17}^\phi}$

notation	constituent link	handcuff graph
$6_{\times}^1 2.1.1.1.1.1$	$\overline{7}_6^2$	$\overline{7}_{33}^{\phi}$
$6_{\times}^1 2.1.1.1.1. - 1$	$\overline{2}_1^2$	$2_1^{\phi} \#_3 \overline{3}_1^{\theta}$
$6_{\times}^1 - 2.1.1.1.1.1$	$\overline{7}_8^2$	$\overline{7}_{36}^{\phi}$
$6_{\times}^1 - 2.1.1.1.1. - 1$	$2_1^2 \#_2 3_1$	$2_1^{\phi} \#_3 \overline{3}_1^{\theta}$
$6_*^1 2.1.1.1.1.1$	0_1^2	$\overline{7}_1^{\phi}$
$6_*^1 - 2.1.1.1.1.1$	0_1^2	$\overline{7}_2^{\phi}$
$6_*^1 1.1.2.1.1.1$	$\overline{2}_1^2$	$\overline{7}_8^{\phi}$
$6_*^1 2 0.1.1.1.1.1$	$\overline{5}_1^2$	$\overline{7}_{16}^{\phi}$
$6_*^1 1.1.2 0.1.1.1$	2_1^2	$\overline{7}_8^{\phi}$
$6_*^1 1.1. - 2 0.1.1.1$	2_1^2	$\overline{6}_2^{\phi}$
$6_*^2 1.2.1.1.1.1$	2_1^2	$\overline{7}_7^{\phi}$
$6_*^2 1.1.2.1.1.1$	0_1^2	$\overline{7}_2^{\phi}$
$6_*^2 1.1.1.2 0.1.1$	2_1^2	$\overline{7}_3^{\phi}$
$6_*^2 1.1.1. - 2 0.1.1$	2_1^2	$\overline{6}_2^{\phi}$
$6_*^3 1.1.1.1.2$	0_1^2	$\overline{7}_2^{\phi}$
$6_*^3 1.1.1.1. - 1. - 2$	2_1^2	$2_1^{\phi} \#_3 4_1^{\theta}$
$6_*^3 1.1. - 1.1.1.2$	0_1^2	$2_1^{\phi} \#_3 4_1^{\phi}$
$6_*^3 1.1. - 1.1. - 1. - 2$	2_1^2	$2_1^{\phi} \#_3 \overline{3}_1^{\theta}$
$6_*^3 1.1. - 1.2 0.1.1$	2_1^2	$2_1^{\phi} \#_3 4_1^{\theta}$
$6_*^3 1.1.1.2 0. - 1. - 1$	2_1^2	$2_1^{\phi} \#_3 4_1^{\theta}$
$6_*^3 1.1.1.1.1. - 2 0$	0_1^2	$\overline{7}_2^{\phi}$
$6_*^3 1.1.1.1. - 1. - 2 0$	0_1^2	$2_1^{\phi} \#_3 4_1^{\phi}$
$6_*^3 1.1. - 1.1.1.2 0$	2_1^2	$2_1^{\phi} \#_3 4_1^{\theta}$
$6_*^3 1.1. - 1.1. - 1.2 0$	2_1^2	$2_1^{\phi} \#_3 \overline{3}_1^{\theta}$
$6_*^4 2.1.1.1.1.1$	2_1^2	$\overline{6}_3^{\phi}$
$6_*^4 - 2.1.1.1.1.1$	2_1^2	2_1^{ϕ}
$6_*^4 2.1. - 1.1.1.1$	2_1^2	$2_1^{\phi} \#_3 \overline{3}_1^{\theta}$
$6_*^4 2. - 1.1.1.1.1$	4_1^2	4_1^{ϕ}
$6_*^4 - 2.1. - 1.1.1.1$	2_1^2	$2_1^{\phi} \#_3 3_1^{\theta}$
$6_*^4 2. - 1.1.1.1.1$	0_1^2	trivial
$6_*^4 2. - 1. - 1.1.1.1$	$\overline{4}_1^2$	$\overline{6}_4^{\phi}$
$6_*^4 - 2. - 1. - 1.1.1.1$	$\overline{5}_1^2$	$\overline{5}_1^{\phi}$
$6_*^4 1.2.1.1.1.1$	0_1^2	$\overline{6}_1^{\phi}$
$6_*^4 - 1.2. - 1.1.1.1$	4_1^2	$\overline{6}_4^{\phi}$
$6_*^4 1. - 2.1.1.1.1$	4_1^2	$\overline{6}_2^{\phi}$
$6_*^4 - 1. - 2. - 1.1.1.1$	5_1^2	5_1^{ϕ}

notation	constituent link	handcuff graph
7_{\times}^3 1.1.1.1.1.1.1	$\overline{7}_6^2$	$\overline{7}_{34}^{\phi}$
7_{\times}^3 1.1.1.1. - 1. - 1. - 1	$\overline{7}_7^2$	$\overline{7}_{35}^{\phi}$
7_{\times}^3 1.1. - 1. - 1.1. - 1.1	$\overline{7}_7^2$	$\overline{7}_{35}^{\phi}$
7_{\times}^3 1.1. - 1. - 1. - 1. - 1. - 1	$\overline{7}_8^2$	$\overline{7}_{36}^{\phi}$
7_*^2 1.1.1.1.1.1.1	0_1^2	$\overline{7}_1^{\phi}$
7_*^2 - 1. - 1.1.1.1.1.1	2_1^2	$2_1^{\phi} \#_3 \overline{3}_1^{\phi}$
7_*^2 - 1. - 1. - 1.1.1. - 1.1	2_1^2	$\overline{7}_9^{\phi}$
7_*^7 1.1.1.1.1. - 1. - 1	0_1^2	$2_1^{\phi} \#_3 2_1^{\phi} \#_3 2_1^{\phi}$
7_*^7 1.1.1.1. - 1.1.1	$\overline{5}_1^2$	$\overline{5}_1^{\phi}$
	4_1^2	6_4^{ϕ}

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